ABSTRACT
The hybrid structure of solid-state pixel detector has a semiconductor sensor that converts the X-ray photons to electric pulses, and an electronics chip that processes and stores the signals. The signals are transferred from the sensor chip to the electronic chip through metallic bump bonds, one per pixel. Image quality parameters have to be assessed in order to estimate the imaging capabilities of the detector. The modulation transfer function (MTF), which describes the spatial resolution of the imaging system, is calculated for the sensor part of the detector. The sensor responses to a point-like source for both mono-energetic and poly-energetic beams are calculated by means of Monte Carlo simulation, in terms of the charge carrier distribution at the collecting surface of the sensor. MTFs for different X-ray spectra, sensor materials and pixel sizes are calculated and compared.

Keywords: modulation transfer function, imaging X-ray detectors, Monte Carlo method.

1. INTRODUCTION
The modulation transfer function (MTF) describes the spatial resolution of a imaging system and it is defined as the modulus of the Fourier transformed response of the imaging device to a pointlike input (delta function). The MTF is a handy descriptor because the stages of system response can be modeled as “filters” and the composite MTF of a system is the product of the MTF of all individual stages. The quality of the acquired image in terms of spatial resolution can be assessed through the MTF, which describe the system response up to, but not including, the stage of sampling [1]. The steps involved for a MTF evaluation are: 1. The sensor response, and 2. The aperture function for further sampling.

The hybrid structure of solid-state pixel detector has a semiconductor sensor that converts the X-ray photons to electric pulses, and an electronics chip that processes and stores the signals. The signals are transferred from the sensor chip to the electronic chip through metallic bump bonds, one per pixel [2]. The sensor responses for different semiconductor materials to mono-directional, mono-energetic and poly-energetic X-ray beams were evaluated in terms of the charge carrier's distribution at the collecting surface of the sensor. The options for the pixel sizes were 100 µm and 250 µm.

The photon transport was simulated by using the Monte Carlo (MC) method. The Poisson equation model was applied to simulate the spread and collection of charge carriers. The sensor responses in the spatial domain were Fourier transformed to obtain the MTFs of the first step. The Fourier transform of the aperture square pixel response is the sinc function (sinc x = (sin x)/x), which was evaluated for different pixel sizes for obtaining the MTFs of the second step. Total MTFs are calculated as the product of the corresponding MTFs of the first and second steps.

2. METHOD
The analyzed sensors are p+-n- abrupt diodes of sensitive volume (bulk) of the materials Si, GaAS, and CdTe. The arrays of p+ implants are the charge collectors. Holes are collected. Sensor thicknesses of 500 µm were assumed for the simulation. The semiconductor material data are presented in Table 1, [3], [4], [5].

<table>
<thead>
<tr>
<th>Data on the semiconductor materials</th>
<th>Si</th>
<th>GaAs</th>
<th>CdTe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permittivity (F/cm²)</td>
<td>1,0E-12</td>
<td>1,2E-12</td>
<td>9,7E-13</td>
</tr>
<tr>
<td>Diffusion coeff. (cm²/s)</td>
<td>37,6</td>
<td>201,2</td>
<td>29,8</td>
</tr>
<tr>
<td>Electrons</td>
<td>11,6</td>
<td>10,4</td>
<td>2,8</td>
</tr>
<tr>
<td>Holes</td>
<td>0,5</td>
<td>0,099</td>
<td>1,45</td>
</tr>
<tr>
<td>Energy to form one e-h pair (eV)</td>
<td>0,1</td>
<td>0,2</td>
<td>0,2</td>
</tr>
<tr>
<td>Fano factor</td>
<td>1450</td>
<td>800</td>
<td>1150</td>
</tr>
<tr>
<td>Mobilities (cm²/V·s)</td>
<td>450</td>
<td>400</td>
<td>110</td>
</tr>
<tr>
<td>Electrons</td>
<td>8</td>
<td>1,0E+05</td>
<td>1,0E+06</td>
</tr>
<tr>
<td>Holes</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The sensor were irradiated with four monodirectional X-ray sources of different energy spectra: a monoenergetic of 30 keV, two X-ray spectra generated at 70 kV with filters of 2.5 mm Al and 3.0 mm Al + 0.01 mm Cu, s1 and s2, and a spectrum generated at 100 kV with a filter of 3.0 mm Al + 0.01 mm Cu, s3. The poly-energetic beams were generated [6].

The Monte Carlo based program GEANT [7] was used for simulating the photon transport in the sensor material. The focal spot of the X-rays generator was assumed to be ideal, that is, a point. The source was treated as a pencil beam. With the sensor perpendicular to the beam direction, the collimated beam was impinging on the sensor's center. The most probable interactions of photons under 100 keV, photoelectric absorption and Rayleigh and Compton scattering were included. The generated number of X-ray photon events was 10^7.

The parameters calculated were the information on the spatial (x, y, z) position and the energy (E, dE) of the photon interactions, irrespective of the particular photon history, for which more than the minimum energy required, to produce a pair of charge carriers, were deposited (dE > 3.6 eV for Si, 4.3 eV for GaAs and 4.7 eV for CdTe). The deposited energy for each interaction provided the amount of charge carriers for GaAs and 4.7 eV for CdTe). The deposited energy for each interaction provided the amount of charge carriers per cluster of charge. The position data provided: first, the location of the charge cluster with respect to the charge formed per cluster of charge. The position data provided: first, the location of the charge cluster with respect to the charge collection electrodes through the x, y coordinates, and second, the distance from the point of charge formation to the p' collectors, through the coordinate z.

The charge collection model assessed the drift time and the diffusion of the signal charge created. The depletion approximation was used, which means that the electric field in the fully depleted semiconductor at the depth coordinate z', E(z'), can be calculated from the Poisson equation [8]:

\[
E(z') = \frac{eN_{\text{bulk}}}{\varepsilon}(z'-d)
\]  

(1)

where:

\(N_{\text{bulk}}\): donor concentration in the semiconductor,
\(\varepsilon\): semiconductor's permittivity,
\(e\): electronic charge,
\(d\): sensor thickness

Since only fully depleted semiconductors are considered, the drift velocity of carriers (holes in this case) as a function of the electric field in the semiconductors is in the linear region, so:

\[
v(z') = \mu \cdot E(z')
\]  

(2)

where:

\(\mu\): mobility of holes.

Drift times are evaluated by integrating the inverse of the drift velocities from the depth point where the charge cluster is produced, which yields [8]:

\[
t(z') = -\frac{\varepsilon}{\mu e N_{\text{bulk}}} \ln \left| 1 - \frac{z'}{(1 + \frac{z'}{d})} \right|
\]  

(3)

a: depletion coeff. defined as \(a = (V_{\text{bi}} - V_{\text{dpl}})/V_{\text{dpl}}\); a is equal to 0 here, since the semiconductors were assumed to be fully depleted.

Drift times were always considered to be much shorter than the carrier lifetimes, so corrections for trapping impurities and recombination due to semiconductor lattice defects were avoided. The density of hole carriers in one dimension can be solved from the diffusion equation in the form [8]:

\[
\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2}
\]  

(4)

where:

\(p\): hole density,
\(D_p\): diffusion constant for holes.

The solution of equation (4) gives that at a given time t, \(p(x)\), is a Gaussian distribution with FWHM of [9]:

\[
\text{FWHM} = 4 \cdot \sqrt{t \cdot D_p \cdot \ln 2}
\]  

(5)

Then, \(\sigma = \text{FWHM}/2.355\) was calculated for the drift times corresponding to each charge cluster. The charge distribution in the collector's plane was calculated by randomly smearing out the x, y position of the charge clusters according to a Gaussian distribution with standard deviation \(\sigma\). The amount of charge for each charge cluster was corrected by applying the Fano factor. In this way the distribution of the collected holes was obtained, representing the response of the sensor to the incidence of a monodirectional beam or delta impulse.

The responses of the sensor for the modeled semiconductor materials resulted in a 2D-charge distribution. The MTF for each sensor was then computed by taking the 2D Fourier transform of the corresponding charge distribution functions. The zero frequency value of the Fourier transform was used to normalize the MTF to 1 at zero frequency.

After X-ray conversion the resulting carriers are collected by the pixel. The spatial integration in a pixel translates into a response that is equal to the collection area of the pixel. The aperture response is 1 inside the active pixel area and 0 outside. The corresponding 2D MTF is [2]:

\[
\sin c(\pi bu) \cdot \sin c(\pi bv) = \frac{\sin(\pi bu) \cdot \sin(\pi bv)}{\pi^2 b^2 u v}
\]  

(6)

where:

\(b\): active length of the pixel, assumed to be a square,
\(u, v\): spatial frequencies in the two dimensions.

The MTFs for the second step were the 2D MTFs for two different active pixel sizes: 250 \(\mu\)m and 100 \(\mu\)m. The pixel aperture responses and their 2D-MTFs are shown in Figure 1.

Fig. 1. Pixel aperture responses for pixel sizes \(px = 100 \mu m\) and \(px = 250 \mu m\), and their 2D-MTFs.
3. RESULTS AND DISCUSSION

The deposited energy, $dE$, and the depth coordinate, $z$, from the photon transport simulation, were used to calculate the amount of charge in the clusters, the drift times of holes to the $p^+$ collectors and the spatial sigma for each of them. 2D histograms containing the number of interactions in the sensor depth coordinate and the deposited energy for these interactions are shown in Figure 2 for a photon energy threshold set to the levels, 3.6 eV, 1.5 keV, 3 keV, and 15 keV. This is for the sensor of Si irradiated with the spectrum $s_1$. The surface of the sensor facing the radiation is positioned in the coordinate $z = 50$ cm, from which the 500 $\mu$m of Si are represented on the depth axis in divisions of 100 $\mu$m.

The distribution of interactions in the depth coordinate didn't show any particular trend, that is, interactions producing charge carriers were occurring over the entire sensor thickness. The largest amounts of interactions occurred for low energy depositions as is seen from the histogram showing all possible interactions with charge production ($dE > 3.6$ eV). Further energy discrimination eliminates low energy deposition events and shows that energies around the mean energy of the photon spectra inside the semiconductor are the most probable depositions. For poly-energetic incident beams, the effect of an energy threshold in reducing the photon scattering is minimal as it has been shown before [10].

The diffusion of holes as a function of the charge drift distance, is presented in Figure 3a for Si at different levels of depletion and for Si, GaAs, and CdTe fully depleted in Figure 3b. The zero depth represents the collection surface; the farther from the collector the charge is produced the larger the diffusion. For a typical Si resistivity of 8 $\Omega\cdot$cm the average drift time of a cluster of holes is of 27 ns. This gives an average FWHM diffusion of 19 $\mu$m, which means a spread around the formation x, y-coordinates with a $\sigma = \pm 8$ $\mu$m. This is valid for fully depleted sensors under the assumption of low electric fields.

For the compound sensor materials, GaAs and CdTe, the common presence of lattice defects and trapping impurities in the bulk was not taken into account. This can make the depletion region smaller than expected, and the drift times larger for shallow impurities, which affects the charge collection times. Also, the amount of charge can be smaller for deep impurities, which leads to a reduction of the signal. However, it was believed that concerning the charge transport, the spread of the charge through the lateral diffusion is the parameter determining the degradation of spatial resolution.

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**Fig. 3. Diffusion FWHM values. a). Si sensor, different levels of depletion: $a=0$, fully depleted, $a=0.5$, 50% over-depleted, $a=1$, 100% over-depleted. b). Fully depleted semiconductors of Si, GaAs and CdTe.**

Figure 4 shows the normalized fitted response in terms of charge in the $p^+$ plane and its MTF, for the Si sensor and the incident spectrum $s_2$. The 10% of the MTF correspond to the resolving power of the system. In general, semiconductor sensors exhibit as high MTF$\text{sensor}$ values as 20% (or more) for spatial frequencies of 5 mm$^{-1}$; and this is the case for the Si sensor. This means that the pure converter can resolve...
between two objects of 100 µm whose center to center distance is 200 µm.

Fig. 4. Sensor response (normalized) in the plane of the charge collection (p+ plane) and its MTF; x - projections on the right side.

In Figure 5a the projections of the converter response MTFs, MTFsensor, are shown for the sensor Si and the different spectra, and in Figure 5b for the spectrum s1 and the different sensor materials. The different incident spectra resulted in very similar MTFsensor for the same sensor material. Slightly higher values of MTFsensor for the entire spatial frequency range were observed for the monoenergetic spectrum. Comparing the MTFsensor of the different sensor materials for the same incident spectra it is seen that the MTFsensor for Si and GaAs are close, while the MTFsensor for CdTe is much smaller in the same frequency range. CdTe has a very low hole mobility, which resulted in a charge spread values of σ almost comparable with the ones for Si. This makes the X-ray response broader and consequently gives smaller MTFsensor at high spatial frequencies.

Total preMTFs are shown in Figure 6 for the three sensors Si, GaAs, and CdTe for the spectrum s1 and the different pixel sizes. The curves are strongly influenced by the shape of the corresponding MTF of the pixel aperture response, making the pure converter MTFsensor to fall drastically for the frequencies above 4 mm⁻¹ for the pixel size equal 250 µm and above 10 mm⁻¹ for the pixel size 100 µm. The preMTFs are plotted until the Nyquist frequencies: 2 mm⁻¹ for the 250 µm pixel, and 5 mm⁻¹ for the 100 µm pixel. Obviously, the smaller the pixel size the higher the preMTF values for high spatial frequencies. For the pixel size 250 µm Si, GaAs and CdTe exhibited preMTFs of 63 %, 62 % and 37 % at the Nyquist frequency 2 mm⁻¹, respectively. For the pixel size 100 µm Si, GaAs and CdTe exhibited preMTFs of 25 %, 20 % and 5 % at the Nyquist frequency 5 mm⁻¹, respectively. In general, smaller thicknesses are desired for GaAs and CdTe

Fig. 5. Sensor’s MTFs. a). MTFsensor for Si and different X-ray photon spectra. b). MTFsensor for the spectrum s1 and the different sensor materials.

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Pre-sampling MTFs, preMTF, different sensor materials and pixel sizes

Fig. 6. Pre-sampling MTFs for the different sensor materials and pixel sizes.
modified depletion thicknesses, lattice defects and presence of traps of carriers for these compound semiconductors should be developed.

4. CONCLUSIONS

A model for evaluating the charge spread through the drift times of the charge produced in semiconductor sensor has been developed with the aid of the MC method. This model should be subsequently improved for the case of compound semiconductors.

The presampling MTFs for three kinds of semiconductor converters were calculated. The MTFsensors showed CdTe as the worse material option, probably due to its low holes mobility. It would perform better with the opposite diode array in which electrons are collected. The MTFsensor results for Si and GaAs were very similar so that’s a good reason to keep choosing Si. On the other hand, depending on the medical application it would need comparatively high photon efficiency and in such a case GaAs will be preferred.

The intrinsic pixel aperture MTFs strongly determined the preMTF values, as expected. By means of simulating the MTFsensor appropriate ways of improving the spatial resolution without changing the size of the pixels can be investigated. Among them, finding the optimal converter thickness, the design of the best grid to account for photon scattering and charge discrimination analysis.

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REFERENCES

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